A STOCHASTIC METHOD FOR THE GENERATION OF OPTIMIZED BUILDING LAYOUTS RESPECTING URBAN REGULATIONS

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ABSTRACT:

In most countries, a project for the development of an urban area has to obey zoning regulations. In France, such zoning regulations are specified in local urban planning schemes (LUPS or PLU in French) defining the right to build at the scale of a parcel. Such rules define, for example, the maximal building height. As the rules are stated in technical documents, they are not easy for non-professionals to comprehend. It is also hard for professionals to assess their impacts. Driven by such issues, we propose to generate 3D building layouts that comply with these rules while optimizing urban indicators (e.g. floor area ratio). A building layout can be seen as a realization of a marked point process (MPP), which is a stochastic model mapping from a probability space to configurations of geometric objects, namely horizontal 3D boxes. Then, the problem of finding an optimized building layout is converted into finding the optimal realization of a MPP of 3D boxes. We solve this optimization problem by trans-dimensional simulated annealing (TDSA), which allows to explore both parameter space and model space in order to find the combination optimizing a given criterion or energy function. A global energy function is defined as the sum of weighted energy terms. Each energy term is able to penalize the building layouts that violate a specific rule or favor the ones according to the optimization task. TDSA generates the optimal building layout by minimizing this global energy using the coupling of a simulated annealing scheme with a Reversible Jump Markov Chain Monte Carlo (RJMCMC) sampler. We studied several common types of the French PLU rules and modeled them into energy terms. A case study is conducted and the results show that our proposed approach is capable of such an optimization task within a short computation time.

1 INTRODUCTION

As urbanization is increasing, environmental, social and economical issues that cities are facing are becoming more and more critical. Thus, in order to consider these issues, urban planners design plans with the aim to regulate city development. These plans are various and differ by their scale (scale of a city or of a building) or by their preoccupations (displacement, social dwellings, etc.). The conception of these documents is a difficult task for two main reasons:

1. the information contained in plans is expressed with free text and their influence on a territory is difficult to assess;
2. as different city issues have complex relationships, the application of a plan supposed to improve an aspect of the city can decrease the performances of other aspects.

To support the design of such documents, it is necessary to provide tools that enable to check if the application of a given document has no side-effect according to the designers' wishes but also regarding other aspects of the city. Among different existing documents, this work is focused on constructability regulation. This kind of regulation is widely defined worldwide and describes constraints that new buildings have to respect to obtain a construction permit. The aim of this article is to propose a robust simulation through the generation of building layouts that respect constraints expressed by these regulations. In France, building regulation is specified in local urban planning schemes (LUPSs or PLU in French). On the one hand, such schemes define the right to build at the scale of a parcel through 2D and 3D morphological rules (maximal height or floor area, for example) that, in particular, projected buildings must respect. On the other hand, PLU documents are opposable, which means that a permit cannot be rejected if a project respects its rules.

In section 2, a state of the art describes several works that simulate similar constructability restrictions. Then, our proposed simulator that is based on the "trans-dimensional simulated annealing " optimization method is presented in section 3. In order to produce desirable building layouts, this simulator integrates constraints from the regulation in its optimization function as described in section 4. Some experimentations are carried out on real world data and with French regulations (section 5).

2 STATE OF THE ART

Different works try to ease the comprehension of urban regulation through several types of approaches by: linking regulation to related geographic features in a 3D viewer (Métral et al., 2009); producing buildable hulls from geometric constraints (El Makchouni, 1987, Murata, 2004, Brasebin et al., 2011); offering the possibility to explore a predefined set of parametric buildings respecting rules (Coors et al., 2009); generating buildings (Turkienicz et al., 2008, Parish and Müller, 2001, Brasebin, 2014) or proposing extensions to existing buildings (Laurini and Vico, 1999). Among these works, building generation offers, in our opinion, the most promising results as it directly provides objects that can be built. Nevertheless, mentioned approaches use heuristics or procedural modeling that do not always fit with every terrain configuration and cannot integrate preferences of different builder-agents. Thus, we found necessary to adapt a building generation method to the specificity of urban regulations.

Building generation methods are explored through various fields such as architecture (Frazer, 1995), urban planning (Rittel and Webber, 1973), geosimulation (Ruas et al., 2011) or computer graphics (Wonka et al., 2003). The goal of the generation differs according to the domain. (Vanegas, 2013) distinguishes geometric and behavior based approaches that are not always discordant. Behavior based approaches aim to produce buildings by integrating human processes and decisions whereas geometric approaches are designed to create fast and visually believable objects. As the objective of this paper is to simulate urban regulations, it is necessary to integrate the preferences of different agents that can construct buildings in order to assess the influence of the rules on different actors (for example, households or building promoters). Generally, these preferences are translated into utility functions that agents try to maximize.

To maximize such functions, optimization methods are used such as Multi-Agent Systems (Ruas et al., 2011) or meta-heuristics like evolutionary algorithms (Frazer, 1995) or simulated annealing (Bao et al., 2013) combined to geometric generative methods like primitive instancing (Perret et al., 2010, Kämpf et al., 2010) or shape grammars (Talton et al., 2011). In order to integrate constraints, a large set of methods and their comparison are described in (Coello Coello, 2010) including rejection, penalization of the optimization function or fixing automatically solutions that do not respect constraints.

3 PROPOSED APPROACH

(Brasebin, 2014) proposes a generic approach to generate building according to a model of French regulations. His method to generate buildings is based on the approach presented in (Tournaire et al., 2010) for extracting building footprints from digital elevation models which relies on marked point processes, a class of random processes which realizations represent a set of objects in a certain parameter space. Whereas in (Tournaire et al., 2010), a configuration represents a collection of rectangle footprints in image space, in (Brasebin, 2014), a configuration consists of rectangular parallelepipeds or cuboids we will from now on refer to as boxes, each representing a building placed inside a land parcel. The author proposes to optimize a given criteria (such as building volume) and a constraint management by rejection during the optimization process. Using rejection allows for genericity in this approach, but might deteriorate generated optimized configurations: the creation of a non connex configuration search space and "barrier effects" due to strict constraints (such as "a building must be aligned with the roads"). Our objective is to propose a more robust simulator, based on (Brasebin, 2014) principles, that overcomes these issues by using an alternative constraints management approach.

3.1 Marked point process

A *marked point process* (Van Lieshout, 2000) is a stochastic model defined by a probabilized space (Ω, π) with $\Omega = \bigcup_{n=0}^{\infty} K^n$ where K denotes the set of possible values of a single object and n the number of such elements in a configuration. A simple probabilization of Ω may be given by the probabilization of K and a discrete probability over $\mathbb{N}$ that samples the number n of objects. Assuming that the probabilization of K may be sampled directly, this yields a direct sampling of Ω according to this so-called reference process. What is needed is however a sampler of Ω according to a target density π that encodes the target objective function to be minimized. We will present, in the next sections, how this may be achieved using Reversible Jump Markov Chain Monte Carlo (RJMCMC) and simulated annealing.

In our problem, each realization of the marked point process is a building layout (a configuration of n buildings). Extending the

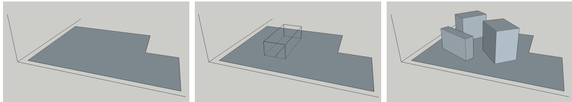


Figure 1: The land parcel, a single building box object and a realization of the marked point process of boxes.

parameterization of 2D rectangles introduced in (Tournaire et al., 2010), each building is described by its 2D center $c_i = (x_i, y_i)$, its 2D semi-major axis vector $\vec{v_i} = (\rho_i, \theta_i)$, its aspect ratio $r_i \leq 1$ and its height h_i, using $r_i = \frac{l_i}{L_i}$ where $L_i = 2\|\vec{v_i}\|$ and l_i are respectively the horizontal box dimensions along and across $\vec{v_i}$.

For efficiency, we prevent the sampling of buildings which centers c_i lie outside the considered land parcel. This generation of centers c_i within the land parcel only is performed using a triangulation of its defining polygon and a parameterization $c'_i = (x'_i, y'_i) \in [0, 1]^2$ that enables the uniform sampling of points inside this set of triangles (Turk, 1990). K is therefore defined as equation 1. Furthermore, r_i is limited to $[r_{min}, 1]$ in order to better control building shapes, h_i is sampled in $[h_{min}, h_{max}]$ with h_{min} the minimum height of a building and h_{max} the maximum height depending of the local planning rules. Figure 1 illustrates the search space as well as a realization of the process.

3.2 Reversible Jump Markov Chain Monte Carlo

RJMCMC is an extension of MCMC that allows the sampling from a configuration space Ω of varying dimension (Green, 1995) that only requires the unnormalized evaluation of its target distribution. This approach consists in using reversible kernels Q_i associated with probabilities $q(i|X_t)$ modeling a random modification of the current configuration X_t to successively propose a new configuration and evaluate its acceptance probability. The algorithm operated by repeating such steps in order to build a series of configurations which stationary distribution is the desired target distribution π. A generic implementation of this algorithm is available and detailed in (Brédif and Tournaire, 2012). In the proposed approach, the reversible kernels used are: birth and death kernel, edge translation kernel, and height scaling kernel.

3.3 Trans-Dimensional Simulated Annealing

In order to drive the RJMCMC sampler to the target probability distribution, whe use simulated annealing (SA), a widely used local metaheuristic (Salamon et al., 2002). This approach of coupling a RJMCMC sampler within a simulated annealing may be referred to as Trans-Dimensional Simulated Annealing (TDSA) (Singh et al., 2008). The idea of the approach is to sample increasingly more selectively from the RJMCMC sampler until a convergence criteria is reached (usually a given number of iterations or a maximum variation of energy during a certain number of iterations).

3.4 Energetic modeling

An attractive property of a marked point process X is that we can define its *probability density function* (pdf) through a Gibbs energy with respect to the reference process: $f(X) = Z^{-1}e^{-E(X)}$, where $Z = \int_X e^{-E(X)} d\mu(X)$ is the normalization factor, with $\mu(.)$ as the probability distribution of the reference process. The energy function $E(X)$ can express the quality of a configuration of marked points. In our case, it indicates the conformity with urban planning rules as well as a desired criteria to optimize (the total building volume for instance). We introduce a global energy composed as the sum of finite weighted energy terms, each of which is formed according to a specific urban planning rule.

$$K = \overbrace{\underbrace{[0,1] \times [0,1]}_{c'_i}}^{point} \times \overbrace{\underbrace{[\rho_{min}, \rho_{max}] \times [0,\pi]}_{\vec{v_i}} \times \underbrace{[r_{min}, 1]}_{r_i} \times \underbrace{[h_{min}, h_{max}]}_{h_i}}^{mark}. \quad (1)$$

Therefore, the problem of finding the optimal building layout that complies with urban planning rules can be translated into finding the configuration of building boxes $\widetilde{X}$ by maximizing the probability $f(.)$ using a RJMCMC sampler under a TDSA framework: $\widetilde{X} = argmaxf(.)$. Urban planning rules vary by countries and cities. The French urban planning scheme is studied in this paper. Common rules are extracted and modeled by energy terms. The detail is presented in the following section.

4 URBAN RULES TO ENERGY TERMS

A PLU (French urban planning scheme) document provides a zoning plan dividing an urban territory into several zones, and specifies applicable regulations for each type of zone. The regulations may include all or some of the 16 articles provided by the urban planning code. Each of these articles describes a fixed theme. For instance, article 1 describes prohibited land use and article 16 describes electronic communication infrastructures and network. This work only focuses on the articles that regulate the spatial aspects of buildings. Therefore, the studied articles are:

- Article 6: building position in relation to public roads,
- Article 7: building position in relation to separative limits,
- Article 8: building position in relation to other buildings,
- Article 9: building footprint,
- Article 10: building height,
- Article 14: floor area ratio.

Articles 6 and 7 can be considered as one theme: building position in relation to parcel borders (front, side, and back borders). Front borders are the borders adjacent to public roads (or at least public space). Side and back borders are separative limits adjacent to private roads or other parcels. Regarding the buildings position, the most common rules are for orientation and location. Therefore, we take into account the following types of rules when generating optimal building layouts:

- Rule A1: distance to parcel borders,
- Rule A2: distance between buildings,
- Rule A3: parcel coverage ratio,
- Rule A4: floor area ratio,
- Rule B1: angle to parcel borders,
- Rule B2: building height.

Rules B1 and B2 can be satisfied by directly setting constraints on geometry parameters ρ and h before MPP sampling if the rules do not depend on dynamic building attributes (e.g location and/or height of itself and/or of its adjacent buildings). For example, if the angle rule is that all buildings should be parallel to a fixed border, which means it does not depend on dynamic building attributes, then the constraint on ρ can be known before sampling. If the angle rule requires a building to be parallel to its nearest border, which means it depends on its own location, then the rule cannot be configured before sampling. Similar for rule B2, the actual rule may have constraints on height differences between adjacent buidings. In such cases, rules B1 and B2 should be modeled by energy functions, along with rules A1 to A4. We discuss the energetic modeling of these rules by categorizing them into three types: unary, binary, and global energy.

4.1 Unary energy

If the calculation of a type of energy for one building does not depend on the attributes of other buildings, we refer to such kind of energy as *unary energy*. Among all types of rules that we studied in this work, rule A1 can be modeled by *unary distance energy* E^{u_d}, and rule B1 can be modeled by *unary angle energy* E^{u_θ}, when it needs energetic modeling. Thus the total *unary energy* can be defined as their weighted sum:

$$E^u = w^{u_d} E^{u_d} + w^{u_\theta} E^{u_\theta} \quad (2)$$

Unary distance energy There are generally three types of borders: front, side, and back borders. One parcel can consist of zero or more than one borders of each type. Regarless of the border type, we refer to the distance from a building to a border as a *unary distance*. A PLU rule for unary distance could be a simple constraint involving only one given constant, for instance, $d < 2m$. It could also depend on external and/or internal parameters. For instance, $d > max(c/2, h)$, where c is the largest width of the road adjacent to the border considered as an external parameter, and h is the building height considered as an internal parameter. The valuation of external parameters can be done before launching the optimization process, whereas the valuation of internal parameters has to be performed during the optimization process. Regardless of the timing of valuation, the left value of the constraint has to be calculated before energetic modeling.

Therefore, all PLU rules for unary distance can eventually be described by the union of finite disjoint real intervals: $d^u_{ij} \in \bigcup_{k=1}^{n} \{I_k(a_k, b_k) | I_k \cap \forall I_{p\neq k} = \varnothing\}$, where d^u_{ij} is the distance from the i^{th} building to the j^{th} border, and I_k is an real interval with endpoints a_k and b_k. The corresponding energy can be defined as:

$$E^{u_d}_{ij} = \begin{cases} 0 & d^u_{ij} \in \bigcup I_k \\ g(d^u_{ij}) & d^u_{ij} \notin \bigcup I_k \end{cases} \quad (3)$$

where $g(.)$ is a penalty function that penalizes non-acceptable distance values.

If the rules for all borders are of logical conjunction: $d^u_{i1} \in A_1 \& d^u_{i2} \in A_2 ... \& d^u_{im} \in A_m$, then the unary distance energy for ith building is $E^{u_d}_i = \sum_{j=1}^{m} E^{u_d}_{ij}$. If there exist rules of logical disjunction like $d^u_{ij} \in A_j || d^u_{ik} \in A_k$, then $E^{u_d}_{i_{jk}} = min(E^{u_d}_{ij}, E^{u_d}_{ik})$. There could also be of logical conjunction in each operand of logical disjunction, so the final $E^{u_d}_i$ can hardly be discribed by a generic formula. Example of such complex rules can be found in section 5. Eventually, the overall unary distance energy for a parcel with n buildings is:

$$E^{u_d} = \sum_{i=1}^{n} E^{u_d}_i \quad (4)$$

Unary angle energy We refer to the angle between a building and a parcel border as *unary angle*. All PLU rules for unary angle can be eventually described by the union of finite disjoint real intervals: $\theta^u_{ij} \in \bigcup_{k=1}^{n} \{I_k(a_k, b_k) | I_k \cap \forall I_{p \neq k} = \varnothing\}$, where θ^u_{ij} is the angle from the ith building to the jth border, and I_k is an real interval with endpoints a_k and b_k. The corresponding energy can be defined as:

$$E^{u_\theta}_{ij} = \begin{cases} 0 & \theta^u_{ij} \in \bigcup I_k \\ g(\theta^u_{ij}) & \theta^u_{ij} \notin \bigcup I_k \end{cases} \tag{5}$$

where $g(.)$ is a penalty function that penalizes non-acceptable angle values.

In practice, a PLU rule for unary angle normally only involves one border, for example, each building should be parallel to a fixed border or to it's nearest border. The involved J^{th} border can be determined before energetic modeling. Therefore, the unary angle energy for the i^{th} building is:

$$E^{u_\theta}_{i} = E^{u_\theta}_{iJ(i)} \tag{6}$$

The overall unary angle energy for a parcel with n buildings is:

$$E^{u_\theta} = \sum_{i=1}^{n} E^{u_\theta}_{i} \tag{7}$$

4.2 Binary energy

If the calculation of a type of energy invloves two buildings in the same parcel, we refer to such kind of energy as *binary energy*. Among all types of rules that we studied in this work, rule A2 can be modeled by *binary distance energy* E^{b_d}, and rule B2 can be modeled by *binary height energy* E^{b_h}, when it concerns the height difference between two adjacent buildings. Thus the total *binary energy* can be defined as their weighted sum:

$$E^{b} = w^{b_d} E^{b_d} + w^{b_h} E^{b_h} \tag{8}$$

Binary distance energy We refer to the distance between two buildings on the same parcel as *binary distance*. All PLU rules for binary distance can be eventually described by the union of finite disjoint real intervals: $d^b_{ij} \in \bigcup_{k=1}^{n} \{I_k(a_k, b_k) | I_k \cap \forall I_{p \neq k} = \varnothing\}$, where d^b_{ij} is the distance between the i^{th} and the j^{th} building, and I_k is an real interval with endpoints a_k and b_k. The corresponding energy can be defined as:

$$E^{b_d}_{ij} = \begin{cases} 0 & d^b_{ij} \in \bigcup I_k \\ g(d^b_{ij}) & d^b_{ij} \notin \bigcup I_k \end{cases} \tag{9}$$

where $g(.)$ is a penalty function that penalizes non-acceptable values. The overall binary distance energy for a parcel with n buildings is therefore:

$$E^{b_d} = \sum_{i=1, j>i}^{n-1, n} E^{b_d}_{ij} \tag{10}$$

Binary height energy We refer to the height difference between two buildings on the same parcel as *binary height*. All PLU rules for binary height can be described by the union of finite disjoint real intervals: $h^b_{ij} \in \bigcup_{k=1}^{n} \{I_k(a_k, b_k) | I_k \cap \forall I_{p \neq k} = \varnothing\}$, where h^b_{ij} is the height difference between the i^{th} and the j^{th} buildings, and I_k is an real interval with endpoints a_k and b_k. The corresponding energy can be defined as:

$$E^{b_h}_{ij} = \begin{cases} 0 & h^b_{ij} \in \bigcup I_k \\ g(h^b_{ij}) & h^b_{ij} \notin \bigcup I_k \end{cases} \tag{11}$$

where $g(.)$ is a penalty function that penalizes non-acceptable values. The overall binary height energy for a parcel with n buildings is therefore:

$$E^{b_h} = \sum_{i=1, j>i}^{n-1, n} E^{b_h}_{ij} \tag{12}$$

4.3 Global energy

If the calculation of a type of energy involves all buildings in the same parcel, we refer to such kind of energy as *global energy*. Among all types of rules that we studied in this work, rule A3 can be modeled by *global coverage energy* E^{g_c}, and rule A4 can be modeled by *global builtup energy* E^{g_f}. Thus the total *global energy* can be defined as their weighted sum:

$$E^{g} = w^{g_c} E^{g_c} + w^{g_f} E^{g_f} \tag{13}$$

Global converage energy is used to model the PLU rule for parcel coverage ratio (PCR). Normally, a constant value $maxpcr$ is given as the upper limit of PCR, and the implicit lower limit of PCR is zero (no building on the parcel). Therefore, the global coverage energy can be defined as:

$$E^{g_c} = \begin{cases} 0 & x_c \in [0, maxpcr] \\ g(x_c) & x_c \in (maxpcr, +\infty) \end{cases} \tag{14}$$

where $g(.)$ is a penalty function that penalizes non-acceptable values.

In order to help urban planners to access the effect of the PLU rule for PCR, an optimization task for PCR is often demanded, which is to generate building layouts with PCR values approching to $maxpcr$. Therefore, an alternative global coverage energy function can be defined as:

$$E^{g_c} = \begin{cases} g_1(x_c) & x_c \in [0, maxpcr] \\ g_2(x_c) & x_c \in (maxpcr, +\infty) \end{cases} \tag{15}$$

where $g_1(.)$ and $g_2(.)$ can be different penalty functions that penalizes less favorable values and non-acceptable values respectfully.

Global builtup energy is used to model the PLU rule for floor area ratio (FAR). Normally, a constant value $maxfar$ is given as the upper limit of FAR, and the implicit lower limit of FAR is zero (no buildings on the parcel). Therefore, the global builtup energy can be defined as:

$$E^{g_f} = \begin{cases} 0 & x_f \in [0, maxfar] \\ g(x_f) & x_f \in (maxfar, +\infty) \end{cases} \tag{16}$$

where $g(.)$ is a penalty function that penalizes non-acceptable values.

Similarly, optimization for FAR is also frequently demanded, and an alternative global builtup energy function can be defined as:

$$E^{g_f} = \begin{cases} g_1(x_f) & x_f \in [0, maxfar] \\ g_2(x_f) & x_f \in (maxfar, +\infty) \end{cases} \tag{17}$$

where $g_1(.)$ and $g_2(.)$ can be different penalty functions that penalizes less favorable values and non-acceptable values respectfully.

5 CASE STUDY

This section presents a case study realized on an actual parcel from the French city of La Courneuve illustrated in figure 2. The objective is to generate optimized building layouts on the given parcel of land with maximal floor area ratio, meanwhile satisfying all the PLU rules concerning this parcel.

Figure 2: The target land parcel for the case study.

The PLU rules for this parcel are:

- Article 6: (1) Distance from each building to the front border should be greater than 3 meters; (2) All buildings should be parallel to the front border,
- Article 7: Distances from each building (with height h) to the two side borders d_1, d_2 should satisfy $(d_1 = 0, d_2 = 0)||(d_1 = 0, d_2 \geq max(6, h))||(d_1 \geq max(6, h), d_2 = 0)$. Distance from each building to the back border should be greater than 4 meters,
- Article 8: Distance between each two buildings should be greater than 4 meters,
- Article 9: Maximal parcel coverage ratio is 0.6,
- Article 10: Minimal building height is 6 meters and maximal building height is 18 meters,
- Article 14: Maximal floor area ratio is 3.

The strategy for handling these rules is:

- Rule A1: Article 6(1) and Article 7 $\rightarrow$ unary distance energy,
- Rule A2: Article 8 $\rightarrow$ binary distance energy,
- Rule A3: Article 9 $\rightarrow$ global coverage energy,
- Rule A4: Article 14 $\rightarrow$ global builtup energy,
- Rule B1: Article 6(2) $\rightarrow$ constraint on parameter ρ,
- Rule B2: Article 10 $\rightarrow$ constraint on parameter h.

5.1 Energy terms

The total energy for each building layout is defined as the weighted sum of unary distance energy e^{u_d}, binary distance energy e^{b_d}, global coverage energy e^{g_c}, and global builtup energy e^{g_f}.

$$e = w^{u_d} e^{u_d} + w^{b_d} e^{b_d} + w^{g_c} e^{g_c} + w^{g_f} e^{g_f} \tag{18}$$

The Gaussian error function erf(.) is used as penalty function to form all the energy terms.

Unary distance energy

1. Distance from i^{th} building to 1^{st} border (front border) should satisfy $d^u_{i1} >= 3$. The energy is defined as:

$$e^{u_d}_{i1} = \begin{cases} -100erf(0.2(d^u_{i1} - 3)) & d^u_{i1} \in (-\infty, 3) \\ 0 & d^u_{i1} \in [3, +\infty) \end{cases} \tag{19}$$

2. Distance from i^{th} building to 2^{nd} and 3^{rd} borders (side borders) should satisfy: $(d^u_{i2} = 0 \,\&\&\, d^u_{i3} = 0) \,||\, (d^u_{i2} = 0 \,\&\&\, d^u_{i3} \geq max(6, h_i)) \,||\, (d^u_{i2} \geq max(6, h_i) \,\&\&\, d^u_{i3} = 0)$. We first define four atomic energy terms:

   - For rule $d^u_{i2} = 0$:

$$e^{u_d}_{i2_a} = \begin{cases} -100erf(0.2d^u_{i2}) & d^u_{i2} \in (-\infty, 0) \\ 100erf(0.2d^u_{i2}) & d^u_{i2} \in [0, +\infty) \end{cases} \tag{20}$$

   - For rule $d^u_{i3} = 0$:

$$e^{u_d}_{i3_a} = \begin{cases} -100erf(0.2d^u_{i3}) & d^u_{i3} \in (-\infty, 0) \\ 100erf(0.2d^u_{i3}) & d^u_{i3} \in [0, +\infty) \end{cases} \tag{21}$$

   - For rule $d^u_{i2} \geq max(6, h_i)$:

$$e^{u_d}_{i2_b} = \begin{cases} -100erf(0.2(d^u_{i2} - R_i)) & d^u_{i2} \in (-\infty, R_i) \\ 0 & d^u_{i2} \in [R_i, +\infty) \end{cases} \tag{22}$$

   where $R_i = max(6, h_i)$

   - For rule $d^u_{i3} \geq max(6, h_i)$:

$$e^{u_d}_{i3_b} = \begin{cases} -100erf(0.2(d_{i3} - R_i)) & d_{i3} \in (-\infty, R_i) \\ 0 & d_{i3} \in [R_i, +\infty) \end{cases} \tag{23}$$

   where $R_i = max(6, h_i)$

   Then the energy for side border rule is defined as:

$$e^{u_d}_{i(2,3)} = min(max(e^{u_d}_{i2_a}, e^{u_d}_{i3_a}), max(e^{u_d}_{i2_a}, e^{u_d}_{i3_b}), max(e^{u_d}_{i2_b}, e^{u_d}_{i3_a})) \tag{24}$$

3. Distance from i^{th} building to 4^{th} border (front border) should satisfy $d^u_{i4} >= 4$. The energy is defined as:

$$e^{u_d}_{i4} = \begin{cases} -100erf(0.2(d^u_{i4} - 4)) & d^u_{i4} \in (-\infty, 4) \\ 0 & d^u_{i4} \in [4, +\infty) \end{cases} \tag{25}$$

Then, the unary distance energy for the i^{th} building is:

$$e^{u_d}_i = e^{u_d}_{i1} + e^{u_d}_{i(2,3)} + e^{u_d}_{i4} \tag{26}$$

And the total unary distance energy for a configuration with n buildings is:

$$e^{u_d} = \sum_{i=1}^{n} e_i^{u_d} \tag{27}$$

Binary distance energy Distance from i^{th} and j^{th} building should satisfy $d_{ij}^b >= 4$. The energy is defined as:

$$e_{ij}^{b_d} = \begin{cases} -100erf(0.2(d_{ij}^b - 4)) & d_{ij}^b \in (-\infty, 4) \\ 0 & d_{ij}^b \in [4, +\infty) \end{cases} \tag{28}$$

Then, the total binary distance energy for a configuration with n buildings is:

$$e^{b_d} = \sum_{i=1, j>i}^{n-1, n} e_{ij}^{b_d} \tag{29}$$

Global coverage energy The parcel coverage ratio should satisfy $x_c \leq 0.6$, so the global coverage energy is defined as:

$$e^{g_c} = \begin{cases} 0 & x_c \in [0, 0.6] \\ 100erf(x_c - 0.6) & x_c \in (0.6, +\infty) \end{cases} \tag{30}$$

Global builtup energy The floor area ratio should satisfy is $x_f \leq 3$, and there is an optimization task for the final building layout approching to the upper limit $x_f = 3$. Thus we define the energy function as:

$$e^{g_f} = \begin{cases} 10(x_f - 3)^2 & x_f \in [0, 3] \\ 100erf(x_f - 3) & x_f \in (3, +\infty) \end{cases} \tag{31}$$

5.2 Implementation and results

The proposed approach is implemented using the open source c++ library librjmcmc (Brédif and Tournaire, 2012), which provides a framework for stochastic optimization using RJMCMC samper and simulated annealing. We use this framework for simulating urban planning rules using a marked point process of cuboids. 50 experiments are conducted on a machine (HP workstation Z210) with 3.3GHz dual core CPU (Intel Core i5-2500) and 8 GB memory under 32 bit Linux environment (Ubuntu 13.04). The average CPU computation time is 252892 ms (approx. 4 minutes and 12 seconds) per experiment with 3 million iterations for simulated annealing.

$$e = 30e^{u_d} + 20e^{b_d} + 20e^{g_c} + 50e^{g_f} \tag{32}$$

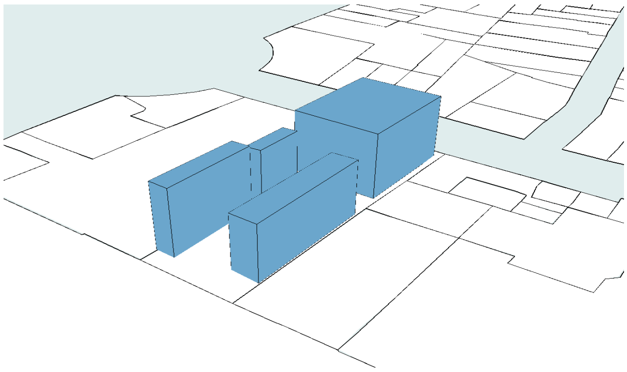
The weights of the energy terms used in these experiments are given in Equation 32. The statistics of the parcel coverage ratio and floor area ratio are given in Table 1. The results can be assessed from two aspects: 1) the optimization of the floor area ratio and 2) the comformity with all the PLU rules reflected by energy values. Therefore, we verify the two results with highest and lowest floor area ratio and the two results with highest and lowest energy.

Table 1: Statistics of 50 experiments.

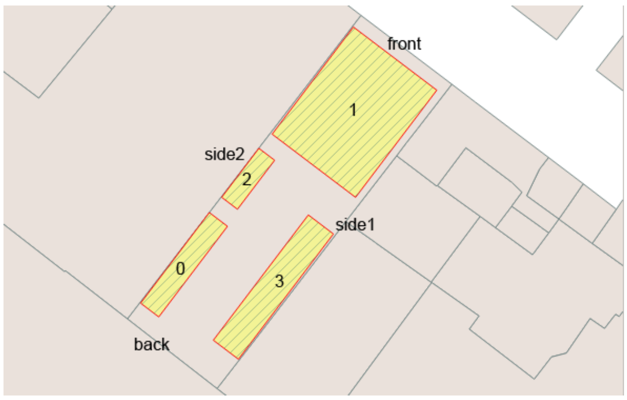
floor area ratio			parcel coverge ratio		
min	max	average	min	max	average
1.60993	2.99672	2.21107	0.310011	0.61312	0.469303

5.2.1 Results with the highest and lowest floor area ratio

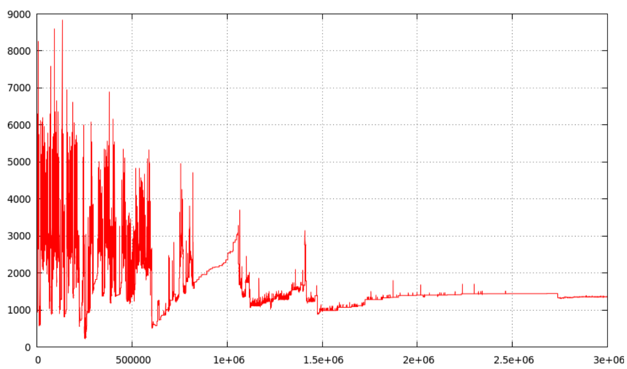
Since the optimization task of this case study is to maximize the floor area ratio within an upper limit $FAR \leq 3$, we verify these two extreme outcomes. The result with the highest floor area ratio is shown in Figure 3, and its detailed properties are given in Table 2. Some rules can be satisfied within acceptable tolerance of error (in blue). There is one violation of the rule (in red) may not be accepted in reality, but it can be easily fixed by postprocessing transformation. The result with the lowest floor area ratio is shown in Figure 4, and its detailed properties are given in Table 3. All the PLU rules are satisfied within 0.2 meters tolerance of error, and the floor area ratio reached to 53.66% of its maximum. This result validates our approach by showing that even the worst possible result can still be satisfying.



(a) Building layout



(b) Footprints



(c) Energy plot

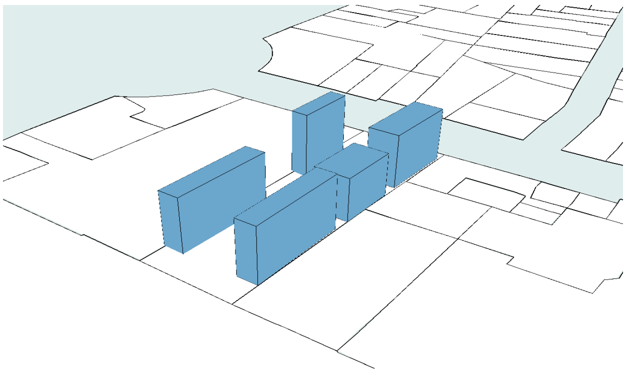
Figure 3: Result with the highest floor area ratio.

5.2.2 Results with the highest and lowest energy

Since the compliance of the PLU rules and the optimization of floor area ratio are reflected by energy, we verify these two extreme outcomes. The result with the highest energy value is shown in Fig-

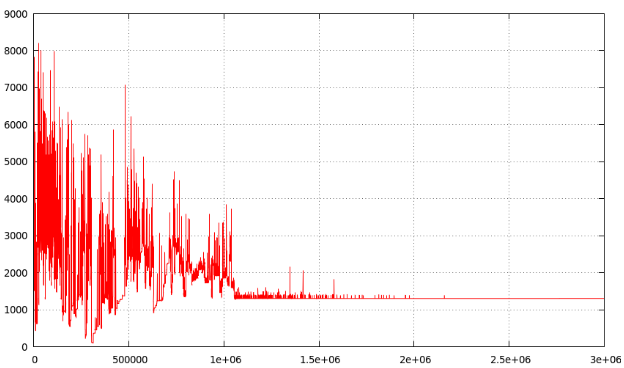
Table 2: Properties of the result with highest floor area ratio.

	dFront	dSide1	dSide2	dBack	height
$bldg_0$	50.3493	18.2548	0.0004	4.1528	18
$bldg_1$	2.8052	1.5012	0.2108	47.5014	18
$bldg_2$	33.9305	18.8940	0.0129	31.1415	15
$bldg_3$	38.6350	0.2542	16.1756	7.2704	15
Binary distance					
d^b_{01}	d^b_{02}	d^b_{03}	d^b_{12}	d^b_{13}	d^b_{23}
20.3253	3.9695	11.6544	3.9066	8.6111	12.1260
Parcel coverage ratio = 0.5230					
Floor area ratio = 2.9967					
Energy					
$30e^{u_d}$	$20e^{b_d}$	$20e^{g_c}$	$50e^{g_f}$	total energy	
1299.39	55.9319	0	0.00538	1355.32	



(a) Building layout

(b) Footprints



(c) Energy plot

Figure 4: Result with the lowest floor area ratio.

ure 5, and its detailed properties are given in Table 4. There is one violation of the rule (in red) may not be acceptable in practice, but can be easily rectified by transformation in postprocessing. The result with the lowest energy is regarded as the best solution. It is shown in Figure 6, and its detailed properties in Table 5. All the PLU rules are satisfied within 0.2 meter's tolerance of error, and the floor area ratio reached to 81.18% of its maximum.

Table 3: Properties of the result with lowest floor area ratio.

	dFront	dSide1	dSide2	dBack	height
$bldg_0$	15.5205	18.8965	6.529e-07	47.1977	18
$bldg_1$	26.7261	0.0796	14.062	36.0455	12
$bldg_2$	2.9182	0.1945	14.9991	54.9106	15
$bldg_3$	45.9468	0.0518	17.8391	7.0341	15
$bldg_4$	45.9698	17.808	0.0003	6.38574	15
Binary distance					
d^b_{01}	d^b_{02}	d^b_{03}	d^b_{04}	d^b_{12}	d^b_{13}
9.98308	11.0886	20.7292	15.6475	3.92361	4.29164
d^b_{14}	d^b_{23}	d^b_{24}	d^b_{34}		
10.1291	23.1443	25.3415	12.8055		
Parcel coverage ratio = 0.329935					
Floor area ratio = 1.60993					
Energy					
$30e^{u_d}$	$20e^{b_d}$	$20e^{g_c}$	$50e^{g_f}$	total energy	
276.123	34.4739	0	966.147	1306.74	

Table 4: Properties of the result with the highest energy.

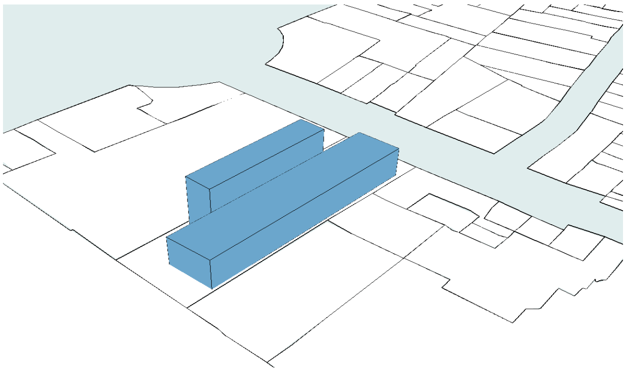
	dFront	dSide1	dSide2	dBack	height
$bldg_0$	7.32	0.558052	9.31182	6.81932	9
$bldg_1$	20.1219	14.4473	0.793807	19.9008	15
Binary distance d^b_{01} = 1.1065					
Parcel coverage ratio = 0.60879					
Floor area ratio = 2.1413					
Energy					
$30e^{u_d}$	$20e^{b_d}$	$20e^{g_c}$	$50e^{g_f}$	total energy	
909.201	1173.75	39.6696	368.681	2491.3	

Table 5: Properties of the result with the lowest energy.

	dFront	dSide1	dSide2	dBack	height
$bldg_0$	3.98306	18.9307	5.364e-06	59.772	18
$bldg_1$	33.7205	0.20625	14.8007	5.28895	15
$bldg_2$	25.1778	13.3745	1.194e-05	5.76251	12
$bldg_3$	2.99916	0.02221	14.2492	51.3647	12
Binary distance					
d^b_{01}	d^b_{02}	d^b_{03}	d^b_{12}	d^b_{13}	d^b_{23}
19.163	7.42981	10.303	5.42371	7.38893	5.08948
Parcel coverage ratio = 0.55157					
Floor area ratio = 2.43554					
Energy					
$30e^{u_d}$	$20e^{b_d}$	$20e^{g_c}$	$50e^{g_f}$	total energy	
155.179	0	0	159.305	314.484	

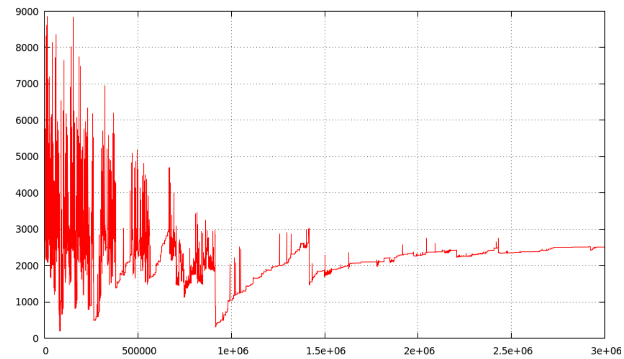
6 CONCLUSION AND PERSPECTIVES

This paper proposes an original approach in order to provide an intuitive understanding of urban planning rules, especially for the rules concerning geometry of the buildings that are allowed to be built. For this purpose, we propose to simulate these rules by generating 3D building layouts in conformity with them, by using a stochastic optimization approach. Moreover, our approach also allows to generate acceptable building layouts with optimized urban indicators (e.g. floor area ratio), so as to help urban planners in the assessment of the impacts of the rules on constructability. Taking a sample of French PLU rules as examples, several common rules are studied and modeled into an energy function. Low energy means less violation of the rules. The minimization of the energy function is realized by stochastic optimization using RJMCMC sampler in a simulated annealing framework. We carried out a case study using real data and rules. The results proved that our method is capable of accomplishing the required task. However, due to the diversity of the rules, the robustness and the genericity of our method are still to be assessed. More complex



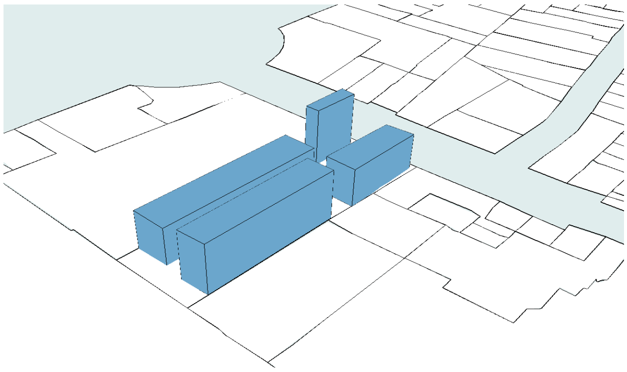
(a) Building layout

(b) Footprints



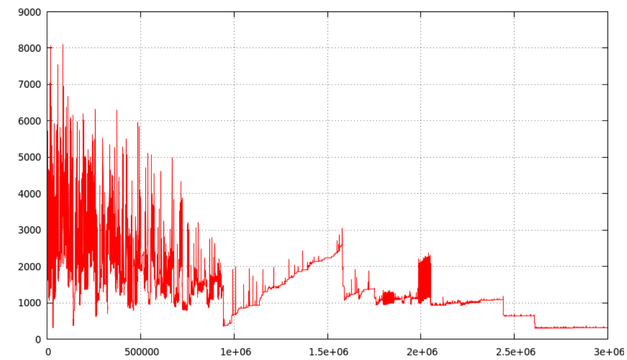
(c) Energy plot

Figure 5: Result with the highest energy.



(a) Building layout

(b) Footprints



(c) Energy plot

Figure 6: Result with the lowest energy.

rules and use cases will be studied in the near future. We plan to lead extra work to design an energy term weight determination method.

This work is realized in the context of the FEDER e-PLU project (http://www.e-plu.fr/) whose aim is to propose a web platform dedicated to territorial engineering. This platform will provide services such as 3D city navigation, right to build consultation and co-design of urban regulations and will be tested on Plaine-Commune inter-communality by the end of 2015.

Some extensions of the simulator will be realized such as the possibility of managing several types of objects associated to projected buildings. This includes objects such as parking spaces, whose dimensions are linked, in the regulation, to the parcel floor area, or building architectural elements, that can be generated with procedural grammars (as it is realized in (Talton et al., 2011)). A last perspective is to link this tool with other simulators that consider other phenomena such as solar radiation or house prices in order to assess the influence of the regulation on these phenomena.

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